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RESEARCH MEMORANDUM

A METHOD FOR DETERMINING CORE DIMENSIONS OF HEAT EXCHANGER WITH ONE DOMINATING FILM RESISTANCE AND VERIFICATION WITH EXPERIMENTAL DATA

By John N. B. Livingood and Anthony J. Diaguila

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

NATIONAL ADVISORY COMMITTEE
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SUMMARY

A procedure is presented for the rapid determination of the core dimensions of a heat exchanger having one dominating film resistance. The length of the exchanger in the direction of the primary fluid flow and the Reynolds number of this flow are determined graphically from three trail solutions of the heat-flow and pressure-drop equations. Methods for determining the other two dimensions are also discussed.

By use of experimental data obtained for a shell and tube liquid-metal-to-air heat exchanger, the calculation procedure presented herein is verified. Results are within the accuracy of the spread of the experimental data. The use of approximate flow conditions yields adequate core dimensions in the examples given herein.

INTRODUCTION

The use of heat exchangers in high-speed, high-altitude aircraft is receiving more and more attention. For such applications, the size and weight of the heat exchanger and the power required to drive the coolants through the exchanger become the predominating factors. In view of this fact, a method for optimizing any one of the heat-exchanger parameters (power, weight, volume, or frontal area) against any other one was developed and reported in reference 1. This optimization was determined for heat exchangers with one dominating film resistance and included seven typical configurations of reference 2.

Recently, an attempt was made to investigate the feasibility of a gas-to-gas heat exchanger for use in reducing the temperature of compressor bleed air prior to its use as the turbine coolant in high-speed, high-altitude turbojet engines. For such an application, the inlet and exit states of both fluids, the available pressure drops of both fluids, the temperature change of one fluid, and the heat capacities of both fluids

may be prescribed. For prescribed pressure drops of both fluids, a conventional calculation procedure for determining the heat-exchanger size becomes lengthy and involved. A method which determines a gas-to-gas heat-exchanger core size with minimum time and effort and which is suitable for the aforementioned application was developed and presented in reference 3 for a prescribed core configuration and a single set of fluid conditions. This procedure has since been generalized for any gas-to-gas heat-exchanger core configuration and a range of fluid conditions in reference 4. The sizes and weights of a number of heat-exchanger cores for possible use in aircraft flying at Mach 2.5 and 70,000 feet were determined by the method of reference 3 and presented in reference 5.

In nuclear reactors, and in some possible aircraft engines, heat exchangers that employ a liquid or liquid metal are of importance. Such exchangers have one dominating film resistance and may be optimized by the method of reference 1. For this type of heat exchanger, the heat exchanged depends essentially on the conditions of the gas (primary coolant). Consequently, only the pressure drop of the primary fluid is considered in this application. The core dimensions of this type of heat exchanger can be determined by use of a modification of the method presented in reference 3. This report presents this modified method and compares, for specified conditions, dimensions determined by use of this method with those of an experimental shell and tube liquid-metal-to-air heat exchanger (ref. 6). The investigation was made at the NACA Lewis laboratory.

SYMBOLS

The following symbols with consistent units are used:

A	heat-transfer area
A'	free-flow area
A_F	frontal area
c_p	specific heat at constant pressure
D	inside diameter of heat-exchanger shell
d	hydraulic diameter
f	friction factor
g	acceleration due to gravity
h	heat-transfer coefficient

k	thermal conductivity
L	heat-exchanger core length
l	half fin length
Pr	Prandtl number
p	pressure
R	gas constant
Re	Reynolds number, $wd/A'\mu$
St	Stanton number, hA'/wc_p
T	temperature
Tu	heat-transfer parameter (number of transfer units, denoted as NTU in ref. 2)
t_f	fin thickness
U	over-all heat-transfer coefficient
V	velocity
v	specific volume
w	weight-flow rate
α	heat-transfer surface area per unit volume
η_f	fin effectiveness
η_T	thermal effectiveness (denoted as ε in ref. 2)
η_O	surface effectiveness
μ	viscosity based on film temperature
ρ	density
σ	ratio of free-flow to frontal area, A'/A_F

Subscripts:

ex	exit

f fin

i inlet

max maximum

min minimum

n no-flow direction

1 heat exchanger side and fluid with finite heat resistance

2 heat exchanger side and fluid with negligible heat resistance

DEVELOPMENT OF METHOD FOR CALCULATING CORE DIMENSIONS

Heat-Exchanger Equations

Under the assumption that one heat resistance is negligible, that is $U = \eta_0 h_1$, the number of transfer units in the heat exchanger may be written as (ref. 1)

$$Tu = \frac{\eta_0 h_1 A_1}{w_1 c_{p,1}} \tag{1}$$

When the heat transfer coefficient h_1 is replaced by its equivalent (eq. (14) of ref. 3),

$$h_1 = \frac{c_{p,1}\mu_1}{d_1} (ReSt)_1 \tag{2}$$

the area A_1 is replaced by its equivalent (eq. (15) of ref. 3),

$$A_1 = \frac{\alpha_1}{\sigma_1} A_1' L_1 \tag{3}$$

and w_1 is expressed in terms of the Reynolds number, equation (1) may be written

$$L_1 = \frac{Re_1 Tu \sigma_1}{\eta_0 \alpha_1 (ReSt)_1} \tag{4}$$

where L_1 is the exchanger core length on the fluid side with the finite heat resistance.

A second expression for the same length L_1 is obtainable from the pressure-drop equation (eq. (17) of ref. 3 with end losses neglected), which may be written

$$L_1 = \frac{d_1}{f_1}\left[\frac{gd_1^2\,\Delta p_1}{Re_1^2\mu_1^2\left(v_{1,ex} + v_{1,i}\right)} - \frac{1 + \sigma_1^2}{2}\,\frac{v_{1,ex} - v_{1,i}}{v_{1,ex} + v_{1,i}}\right] \tag{5}$$

where

$$v_{1,i} = \frac{RT_{1,i}}{p_{1,i}}$$

$$v_{1,ex} = \frac{R\left(T_{1,i} + \Delta T_1\right)}{p_{1,i} - \Delta p_1}$$

and μ is evaluated at the film temperature.

For prescribed values of $p_{1,i}$, $T_{1,i}$, Δp_1, ΔT_1, $w_1c_{p,1}$, $w_2c_{p,2}$, and $T_{2,i}$, and for a prescribed core configuration, equations (4) and (5) become a pair of equations in two unknowns, Re_1 and L_1. A method of solution for these equations will be given later. For the conditions previously prescribed, values of the other dimensions L_2 and L_n of the heat-exchanger core can be determined in the following way. With the values of Re_1 and L_1 obtained from the solution of equations (4) and and (5), the frontal area for the primary coolant side (or the product L_2L_n) can be determined from the continuity equation for the primary fluid, that is,

$$w_1 = \sigma_1 Re_1 \frac{\mu_1}{d_1} L_2L_n \tag{6}$$

If either L_2 or L_n is known or calculable, the other length (L_n or L_2) can be determined from equation (6). Three cases will be considered.

Case 1. - For some applications of the type of heat exchanger considered herein, installation or other considerations may require that either L_2 or L_n be restricted in size. In this case, the other length can be determined directly from equation (6).

Case 2. - In other applications, it is conceivable that the velocity of the secondary fluid may be restricted to a certain value. In this

case, the value of L_n can be determined from the continuity equation of the secondary fluid written in the following form:

$$w_2 = \rho_2 V_2 \sigma_2 L_1 L_n \tag{7}$$

With the appropriate substitutions, L_2 can then be found from equation (6).

Case 3. - If neither of the limitations of cases 1 or 2 apply, it may be necessary to assume a length for either L_2 or L_n, and then obtain a length for the other dimension (L_n or L_2) from equation (6). In this way a series of heat-exchanger geometries can be obtained, and the particular selection is left to the designer.

Calculation Procedure for Solving Equations (4) and (5)

From the prescribed conditions stated previously and from the heat-flow equation

$$w_1 c_{p,1} \Delta T_1 + w_2 c_{p,2} \Delta T_2 = 0 \tag{8}$$

the value of ΔT_2 is obtained. The values of c_p are based on bulk temperature. The equation

$$\eta_T = \left| \frac{\Delta T_{max}}{T_{1,i} - T_{2,i}} \right| \tag{9}$$

may then be solved for η_T. For various flow conditions, reference 2 presents plots of η_T against Tu with $(wc_p)_{min}/(wc_p)_{max}$ (or $w_1 c_{p,1}/w_2 c_{p,2}$ for the cases considered herein) as parameter. From the prescribed conditions, equation (9), and the appropriate curve in reference 2, the value of Tu is then determined.

The values of α_1 and σ_1 are obtainable from the prescribed core configuration; and η_O for either side is obtained from the following expressions (eqs. (12) and (13) of ref. 3):

$$\eta_O = 1 - \frac{A_f}{A}(1 - \eta_f) \tag{10}$$

where

$$\eta_f = \sqrt{\frac{kt_f}{2h}}\,\frac{1}{l}\,\tanh\sqrt{\frac{2h}{kt_f}}\,l \tag{11}$$

Friction and heat-transfer data $[f_1$ and $(ReSt)_1$ for use in equations (5) and (4)$]$ in terms of the Reynolds numbers are obtainable from reference 2 for various core configurations or from equations applicable to the type of passages considered in the selected core.

From the preceding information, equations (4) and (5) can each be solved for L_1 for a series of assumed values of Re_1. The intersection of the curves representing the corresponding values of Re_1 and L_1 from the two equations yields the desired correct values of Re_1 and L_1.

VERIFICATION OF METHOD BY USE OF EXPERIMENTALLY DETERMINED VALUES OF Tu

The method for calculating the core dimensions of a heat exchanger presented herein may be verified with the aid of experimental data obtained for a liquid-metal-to-air shell and tube heat exchanger (ref. 6). A schematic diagram of this experimental exchanger is shown in figure 1. Air flowed through 241 tubes of 3/16-inch outer diameter, 0.016-inch wall, and 28-inch length ($L/d = 180$). A 4.25-inch-inside-diameter shell surrounded the tube bundle. Sodium flowed over the tubes as indicated in figure 1. For this exchanger, α_1 is 99.26 feet^{-1} and σ_1 is 0.3205. Experimental data necessary for the verification are presented in table I.

Friction and heat-transfer data were correlated in reference 6 by use of appropriate and well-established correlations. These same correlations will be used herein. The conventional single-tube heat-transfer correlation corrected for an L/d ratio of 180 and with an assumed value of $Pr^{2/3}$ equal to 0.75 will be employed; this correlation is

$$(ReSt)_1 = 0.028\,Re_1^{0.8} \tag{12}$$

The Kármán-Nikuradse friction correlation

$$\frac{1}{\sqrt{8\frac{f_1}{2}}} = 2\log\left(Re_1\sqrt{8\frac{f_1}{2}}\right) - 0.8 \tag{13}$$

is also used.

For any set of experimental data, the value of $(ReSt)_1$ can be obtained from equation (12) for the experimentally determined Re_1. With these values, Re_1 and $(ReSt)_1$, the experimental exchanger length L_1 of 28 inches, the values of α_1 and σ_1 for this exchanger, and η_0 of 1 (there are no fins), the correct value of Tu for this set of data can be obtained from equation (4). The verification of the method of calculating the heat-exchanger core dimensions presented herein can be demonstrated by use of these experimentally determined values of Tu as follows.

For any set of data, and the corresponding experimental value of Tu, equation (4) can be solved for L_1 for three assumed values of Re_1. The values of $(ReSt)_1$ corresponding to the assumed values of Re_1 are obtained from equation (12). For the same three assumed values of Re_1, values of f_1 are obtained from equation (13). With these values and the experimentally determined values of $p_{1,i}$, Δp_1, $T_{1,i}$, and ΔT_1, three solutions of equation (5) can be determined. The intersection of the curves representing the solutions of equations (4) and (5) yields the desired values of L_1 and Re_1. Any discrepancy between this value of L_1 and the design length of 28 inches must therefore be attributed to (1) the discrepancies between the heat-transfer and friction equations (eqs. (10) and (11)) and the measured heat-transfer and friction data, and (2) the differences in the heat flow between the two fluids (about 8%, according to ref. 6).

Since the experimental exchanger is of the tube-shell type, the frontal area on the primary fluid side is circular in shape, and equation (6) may therefore be written

$$w_1 = \sigma_1 Re_1 \frac{\mu_1}{d_1} \frac{\pi D^2}{4} \tag{14}$$

where the frontal area $L_2 L_n$ is replaced by $\pi D^2/4$. Equation (14) can be solved for D, and these values of D can be compared with the design dimension of 4.25 inches.

APPLICATION OF METHOD FOR DESIGN PURPOSES (WHEN EXPERIMENTAL DATA ARE UNAVAILABLE)

When one is faced with the problem of designing a heat exchanger and experimentally determined values of Tu are not available, it is necessary to obtain values of Tu by use of equation (9) and an

appropriate relation between η_T, $(wc_p)_{min}/(wc_p)_{max}$ (equal to $w_1c_{p,1}/w_2c_{p,2}$ herein), and Tu for the particular flow conditions involved. If such flow conditions are well defined (such as crossflow, counterflow, or parallel flow), accurate values of Tu are available from reference 2. If flow conditions are not well defined, it may be necessary to approximate the values of Tu by use of certain assumed flow conditions. For the experimental heat exchanger previously described, crossflow-counterflow conditions prevail. Since reference 2 does not contain a Tu against η_T plot for this flow condition, it was necessary to assume an approximating condition; crossflow was assumed. When design conditions identical to the inlet and exit conditions of the experimental exchanger are considered and when values of Tu for assumed crossflow are applied, approximate core dimensions can be determined. These approximate dimensions are compared with those of the experimental exchanger. It should be emphasized that the calculation procedure is applicable for any chosen set of inlet and exit conditions. The use of experimentally measused conditions herein is made solely for the purpose of comparison. In this way, it is possible to determine whether the use of approximate flow conditions results in calculated dimensions close to those obtained with the use of true flow conditions.

From the experimental inlet and exit conditions and the assumed crossflow condition, values of Tu for each set of data can be obtained. Values of η_T are found from equation (9). For these η_T values and the corresponding values of $(wc_p)_{min}/(wc_p)_{max}$ (determined from the information presented in table I), the Tu values are read from figure 2; figure 2 is reproduced from reference 2 and applies for crossflow conditions. For three assumed values of Re_1, and the corresponding values of $(ReSt)_1$ and f_1 obtained from equations (12) and (13), three solutions for L_1 of each of equations (4) and (5) can be determined for each set of data in table I. The intersection of the curves through the three solutions of each of equations (4) and (5) gives the desired values of L_1 and Re_1.

RESULTS AND DISCUSSION

Initial calculations were made according to the procedure discussed in the section VERIFICATION OF METHOD BY USE OF EXPERIMENTALLY DETERMINED VALUES OF Tu. The results of these calculations are as follows:

Run	Experimentally determined Tu	Re_1	L_1, in.	D, in.	$\pi D^2/4$, sq in.	Volume, cu in.
1	2.58	31,750	28.6	4.27	14.3	408
2	3.05	14,950	29.1	4.11	13.3	387
3	2.72	25,700	28.8	4.24	14.1	406
4	2.48	33,000	27.8	4.51	16.0	445
5	2.67	24,700	28.2	4.37	15.0	424
6	2.71	22,000	27.7	4.36	14.9	413
7	3.07	13,400	28.6	4.10	13.2	377
8	2.33	43,000	27.5	4.54	16.2	446
9	2.29	45,300	27.4	4.58	16.5	451

Figure 3(a) shows the solutions of equations (4) and (5) for the assumed values of Re_1, and the intersection point of the curves joining these solutions (L_1 and Re_1) for run 5. Comparison of the average calculated values of L_1, $\pi D^2/4$, and volume for the nine runs (28.2 in., 14.8 sq in., and 417 cu in., respectively) with those of the experimental exchanger (28 in., 14.2 sq in., and 397 cu in., respectively) resulted in discrepancies of about 1, 4, and 5 percent, respectively. For some runs, these discrepancies increased to as much as about 4, 16, and 13 percent, respectively. Reference 6 shows that the measured data deviated from the heat-transfer and friction correlation equations by as much as 15 percent. Since the correlation equations were used in the calculations just discussed, it may be concluded that the calculation method presented herein is verified by giving results within the accuracy of the experimental data.

Calculations were also made by the procedure described in APPLICATION OF METHOD FOR DESIGN PURPOSES (WHEN EXPERIMENTAL DATA ARE UNAVAILABLE). As stated previously, inlet and exit conditions identical to the experimental values listed in table I were selected, but crossflow was assumed as an approximation for the flow conditions within the exchanger. The calculations, which for this case may be termed approximate because of the assumed crossflow, were made for the same trial values of Re_1 that were assumed for the other calculations. The results are

Run	Cross-flow, Tu	Re_1	L_1, in.	D, in.	$\pi D^2/4$, sq in.	Volume, cu in.
1	2.40	31,900	28.1	4.26	14.1	400
2	2.52	16,100	24.7	3.96	12.3	305
3	2.30	27,800	24.9	4.02	12.7	316
4	2.18	34,400	24.8	4.41	15.3	381
5	2.41	25,600	25.7	4.30	14.5	372
6	2.65	22,200	27.5	4.34	14.7	406
7	2.85	13,800	26.9	4.03	12.8	345
8	2.16	44,200	25.6	4.49	15.8	403
9	2.12	46,500	25.6	4.52	16.0	411

Figure 3(b) presents the solutions of equations (4) and (5) for the assumed values of Re_1 and the intersection point of the curves joining these solutions for run 5. Average values of L_1, $\pi D^2/4$, and volume for the nine runs (26 in., 14.2 sq in., and 371 cu in., respectively) now differ from the design values by about 7, 0, and 6 percent, respectively. For some runs, discrepancies increased to as much as 12, 13, and 23 percent.

From the calculations presented, it may be concluded that the use of approximate flow conditions yields adequate core dimensions. For precise calculations, an accurate knowledge of flow conditions is essential.

SUMMARY OF RESULTS

The results of this investigation are summarized as follows:

1. A method is presented for the rapid determination of the core dimensions of a heat exchanger having one dominating film resistance. Three trial solutions of the heat-flow and pressure-drop equations are sufficient for determining the heat-exchanger length in the direction of the primary fluid flow and the Reynolds number of this flow. Methods for determining the other two core dimensions are also discussed.

2. The method is verified with experimental results obtained from a shell and tube liquid-metal-to-air heat exchanger. For experimentally determined values of the heat-transfer parameter, the average values of the exchanger core length (on the fluid side with the finite heat resistance), frontal area, and volume differed from the experimental exchanger values by about 1, 4, and 5 percent, respectively. When the heat-transfer parameter values were found from an available heat-transfer chart for flow conditions that approximated those existing in the exchanger, the average values of the exchanger core length (on the fluid side with the finite heat resistance), frontal area, and volume differed from the experimental exchanger values by about 7, 0, and 6 percent, respectively.

3. The discrepancies between the calculated dimensions and those of the experimental heat exchanger, when utilizing the experimentally determined values of the heat-transfer parameter, result from the scatter in the heat-transfer and friction data as well as the apparent differences in the heat flow between the two fluids.

4. For the calculations considered herein, the use of approximate flow conditions gave adequate core dimensions. An accurate knowledge of flow conditions is essential for the determination of precise core dimensions.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, November 23, 1956

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TABLE I. - EXPERIMENTAL DATA OBTAINED IN SHELL AND TUBE LIQUID-METAL-TO-AIR HEAT EXCHANGER

Run	w_1, lb/sec	$T_{1,i}$, °F	$T_{1,ex}$, °F	$-\Delta T_1$, °F	$T_{1,f}$, °F	$p_{1,i}$, lb/sq ft	$p_{1,ex}$, lb/sq ft	Δp_1, lb/sq ft	w_2, lb/sec	$T_{2,i}$, °F	$T_{2,ex}$, °F	ΔT_2, °F
1	1.631	53	663	610	532	5415	2693	2722	2.270	848	565	283
2	.781	61	830	769	665	3405	2240	1165	5.240	927	841	86
3	1.337	58	719	661	586	4679	2181	2528	5.362	845	721	124
4	1.966	306	728	422	644	6526	2140	4386	5.438	830	713	117
5	1.482	295	865	570	744	5382	2094	3288	5.366	970	848	122
6	1.375	619	964	345	886	5392	2179	3213	5.144	1018	943	75
7	.762	599	1023	424	924	3480	2011	1469	6.111	1061	1015	46
8	2.667	508	784	276	730	8746	3104	5642	5.224	865	762	103
9	2.833	600	764	164	732	9135	3211	5924	5.283	815	751	64

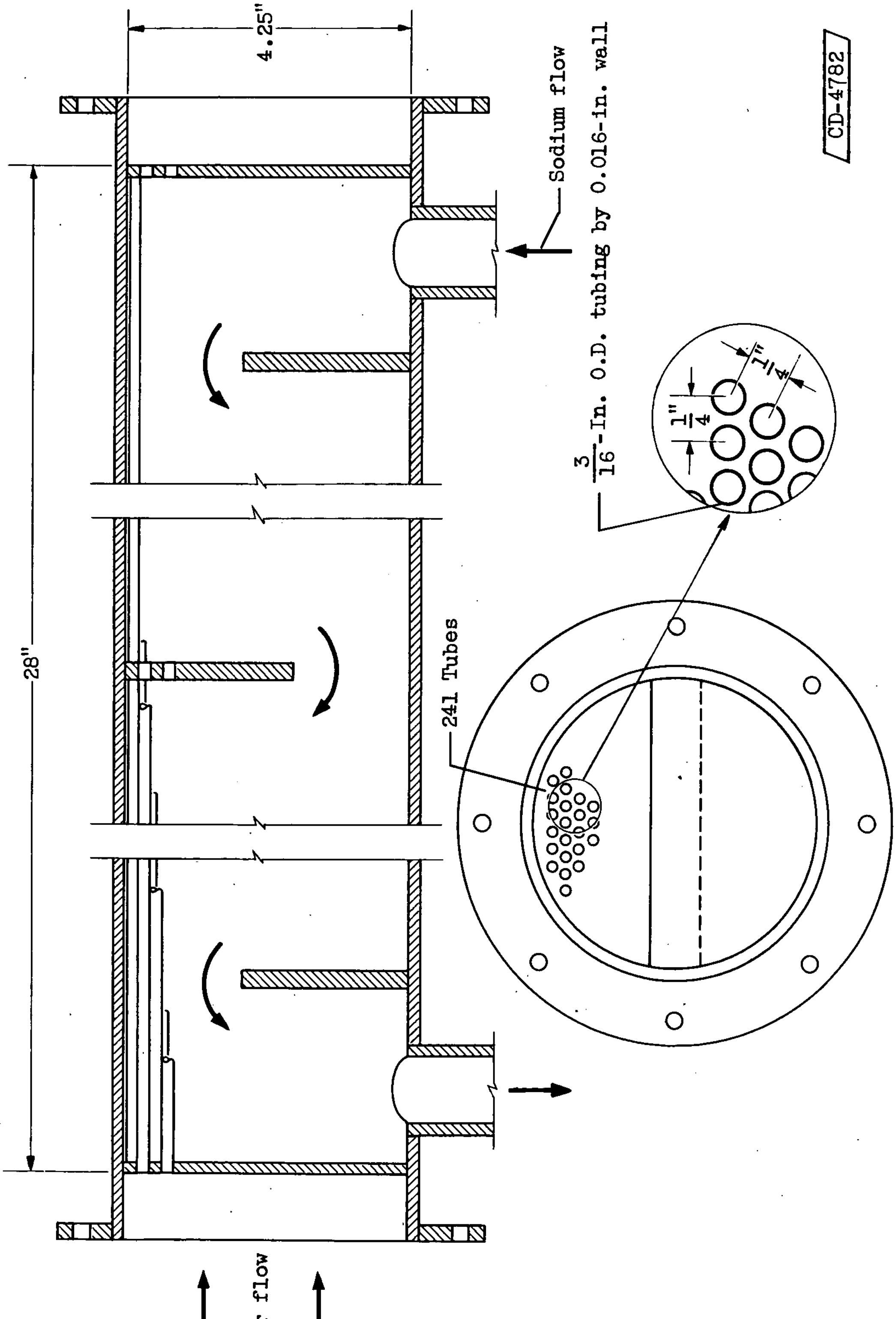


Figure 1. - Schematic diagram of shell and tube heat exchanger (ref. 6).

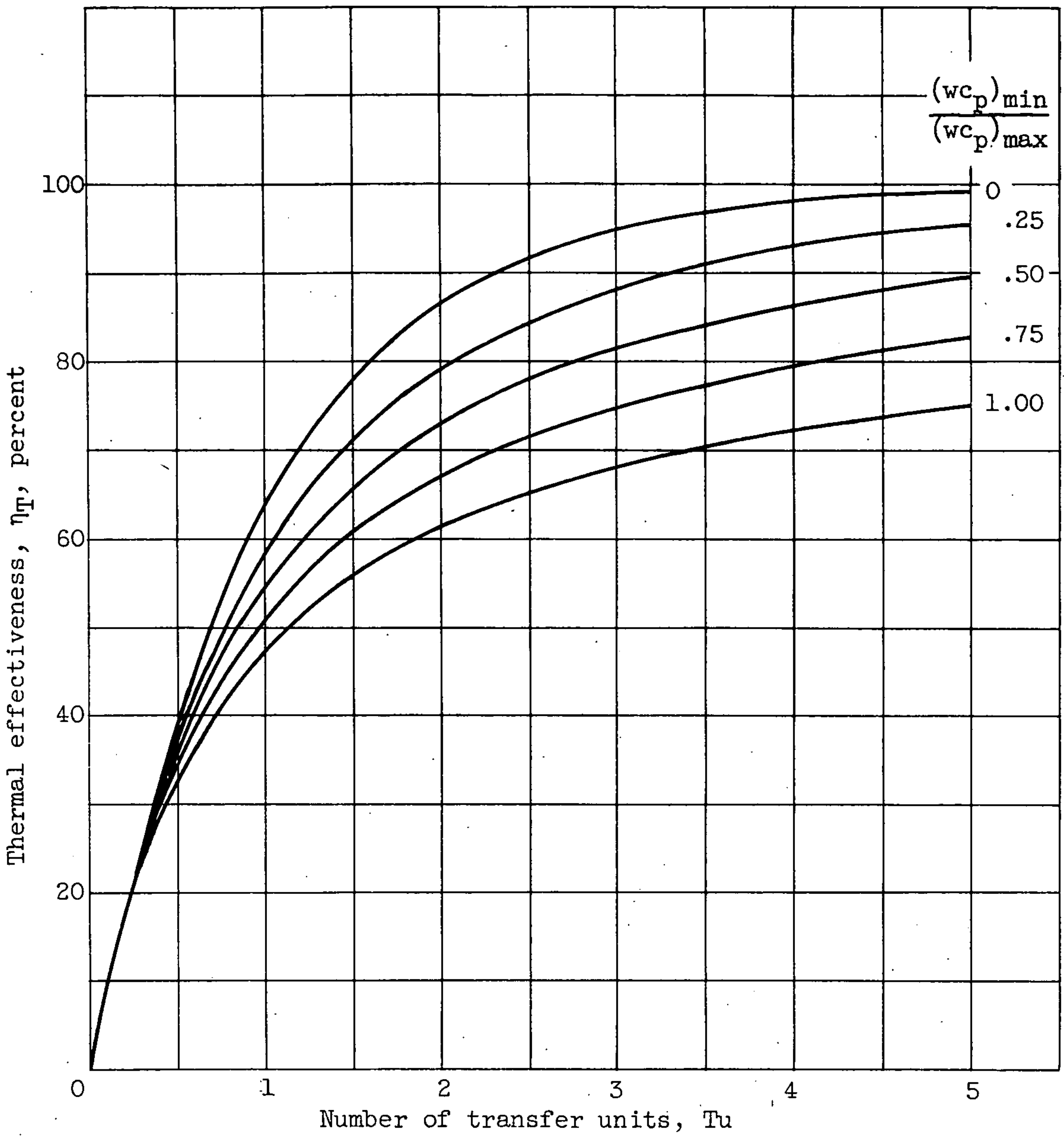
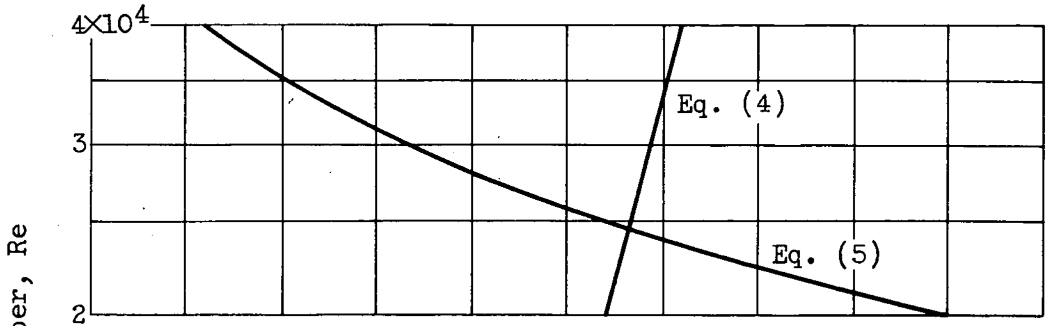
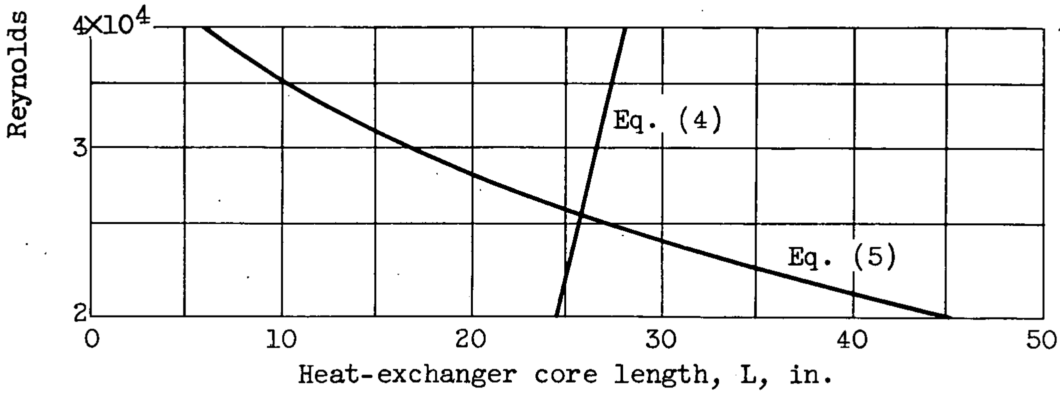


Figure 2. - Performance of crossflow heat exchanger with fluids unmixed (ref. 2).



(a) Experimental heat-transfer parameter.



(b) Crossflow heat-transfer parameter.

Figure 3. - Solution of equations (4) and (5) for run 5.